Starter

10 Solve these equations for $-180° \leq \theta \leq 180°$. Show your working.

a $4\sin\theta = 3\cos\theta$

b $4\sin\theta = 3\tan\theta$

a

$$\frac{\sin\theta}{\cos\theta} = \frac{3}{4}$$

$$\tan\theta = 0.75$$

$$\theta = 36.9° \text{ or } -180° + 36.9° = -143.1°$$

b

$$4\sin\theta = \frac{3\sin\theta}{\cos\theta}$$

$$4\sin\theta\cos\theta - 3\sin\theta = 0$$

$$\sin\theta(4\cos\theta - 3) = 0$$

$$\sin\theta = 0 \text{ or } \cos\theta = \frac{3}{4}$$

$$\theta = 0°, \pm 180° \text{ or } \theta = \pm 41.4°$$

R Forces and Newton's laws

R1 Understand the concept of a force; understand and use Newton's first law.

Assessed at AS and A-level

Teaching guidance

Students should:

- understand types of force, including:
  - normal reaction force
  - tension in a string or a rod
  - thrust in a rod
  - weight
  - friction.
- know that the resultant force acting on a body is zero if a body is in equilibrium
- be able to find unknown forces acting on bodies that are at rest or moving with constant velocity
- be able to model forces as vectors
- be able to find the resultant of several forces acting at a point.

8.1 Forces

When an object is moved, then a force has been applied to it.

Force is a vector quantity as it has magnitude (measured in Newtons, N) and direction (given as a bearing or an angle).

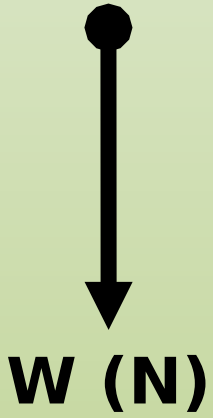
Force can also be given in component form,
e.g. or

8.1 Forces

Common Forces Acting on an Object

Weight:

The force on an object due to the gravitational pull of the earth. Modelled vertically downwards.



W = mass(kg) x g

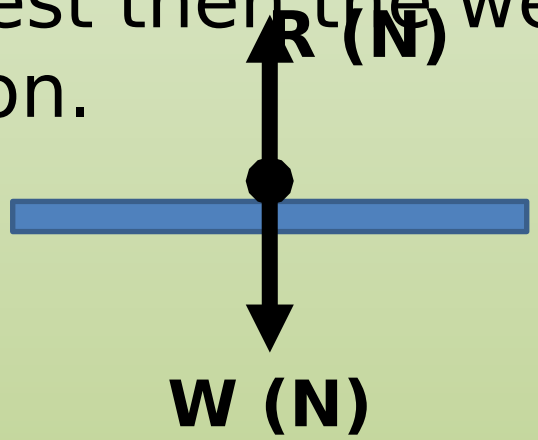
W = mg

8.1 Forces

Common Forces Acting on an Object

Normal Reaction:

This is a contact force which pushes upwards to counteract the weight of an object on a surface. The normal reaction is always perpendicular (normal) to the surface on which the object is placed. If the object is at rest then the weight is equal to the normal reaction.



8.1 Forces

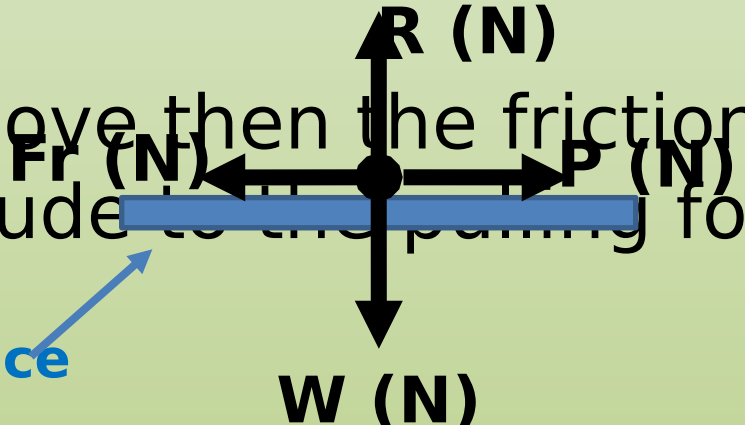
Common Forces Acting on an Object

Friction:

This is a contact force which appears due to the rough surfaces trying to move relative to each other. There is no friction between surfaces described as *smooth*.

Friction opposes the intended direction of motion.

If the object does not move then the frictional force is equal in magnitude to the pulling force.

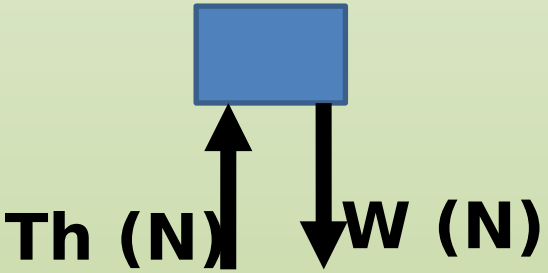


8.1 Forces

Common Forces Acting on an Object

Thrust:

If an object is pushed upwards by a rod, then the force provided by the rod, which opposes the weight, is known as the thrust.

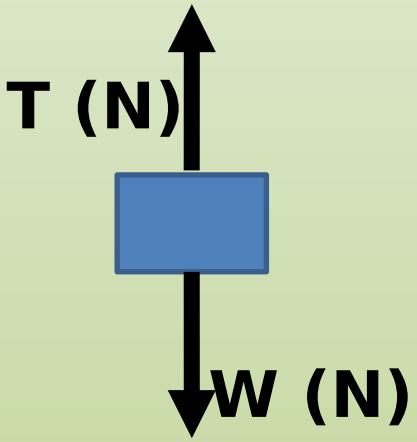


8.1 Forces

Common Forces Acting on an Object

Tension:

If an object is pulled upwards by a string, the force in the string is tension.



8.1 Forces

Newton's 1st Law states that:
An object will remain at rest or continue to move with constant velocity () unless an external force is applied to it.

When a particle is in equilibrium, the resultant force, in any direction will be zero.

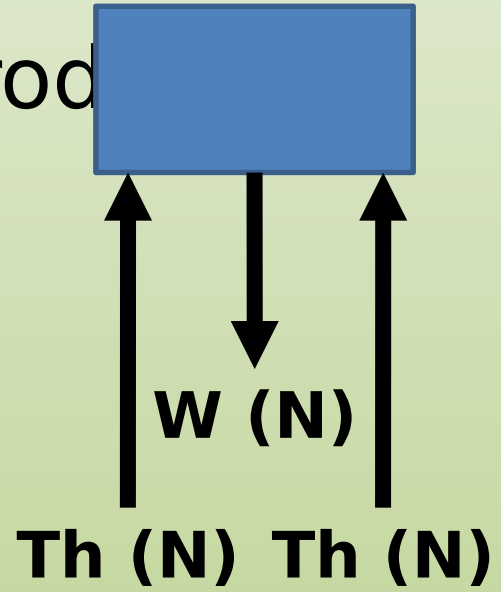
If the object is not in equilibrium, the resultant force will not be zero and the object will accelerate in the direction of

8.1 Forces

Example 1a

Draw the forces acting on a box in each of the following situations:

a) Pushed upwards by two rod

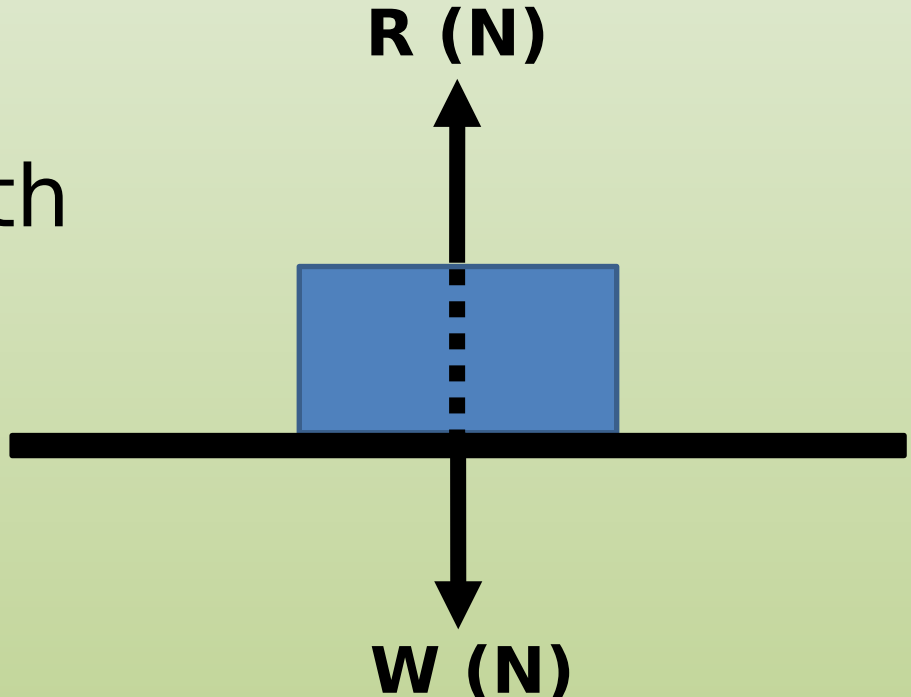


8.1 Forces

Example 1b

Draw the forces acting on a box in each of the following situations:

b) Resting on a smooth horizontal surface

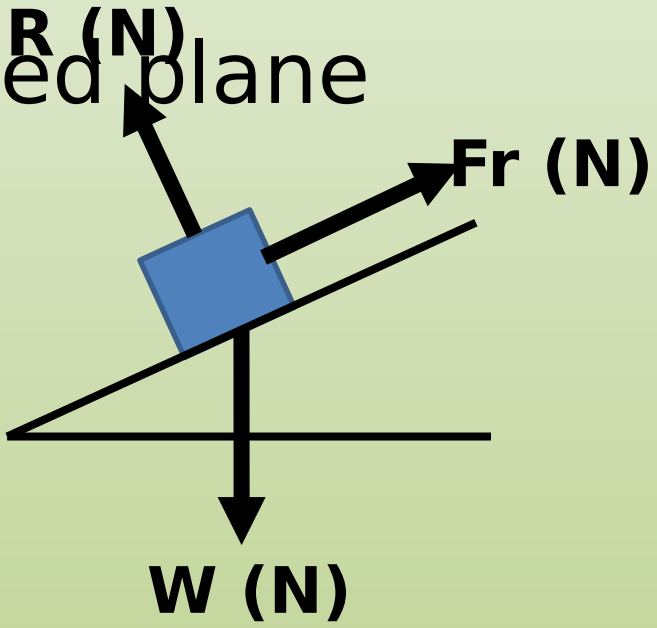


8.1 Forces

Example 1c

Draw the forces acting on a box in each of the following situations:

c) Resting on a rough inclined plane



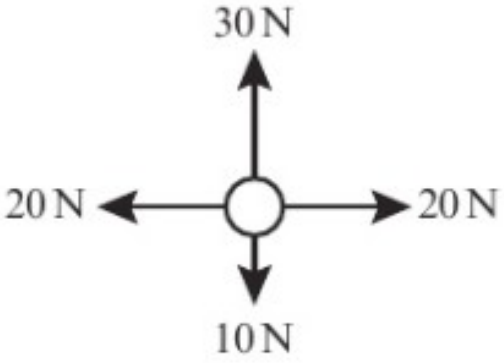
8.1 Forces

Example 2a

The diagram shows the forces acting on a particle.

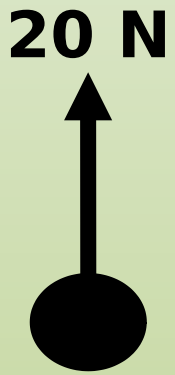
a Draw a force diagram to represent the resultant force.

b Describe the motion of the particle.



Resolving horizontally ⇔ :
so the forces are balanced.

Resolving vertically ↑:
so the resultant force is 20N upwards



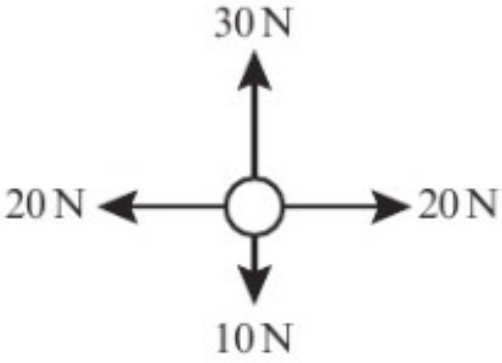
8.1 Forces

Example 2b

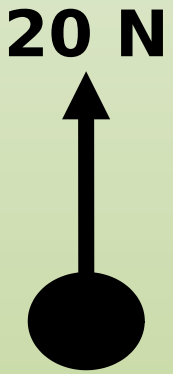
The diagram shows the forces acting on a particle.

a Draw a force diagram to represent the resultant force.

b Describe the motion of the particle.



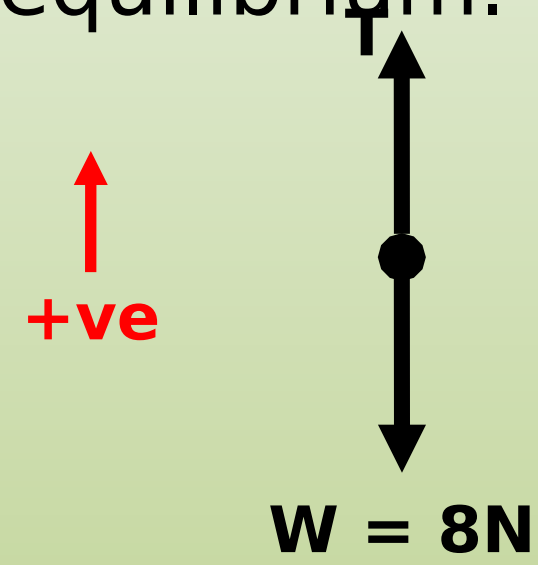
The particle is accelerating upwards.



8.1 Forces

Example 3

An object has a weight of 8N. It is suspended by a string and is in equilibrium. Find the tension in the string.



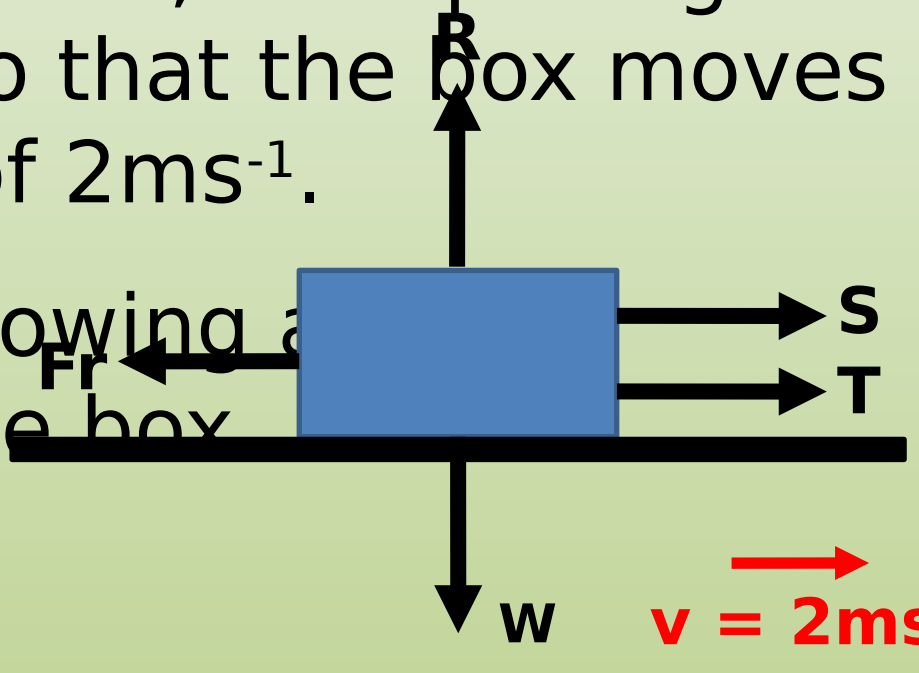
Resolving vertically ↑ :

8.1 Forces

Example 4a

A box of weight W rests on a rough horizontal table. It is pulled by two horizontal strings T and S, both pulling in the same direction, so that the box moves at a constant speed of $2ms^{-1}$.

a) Draw a diagram showing a
forces acting on the box

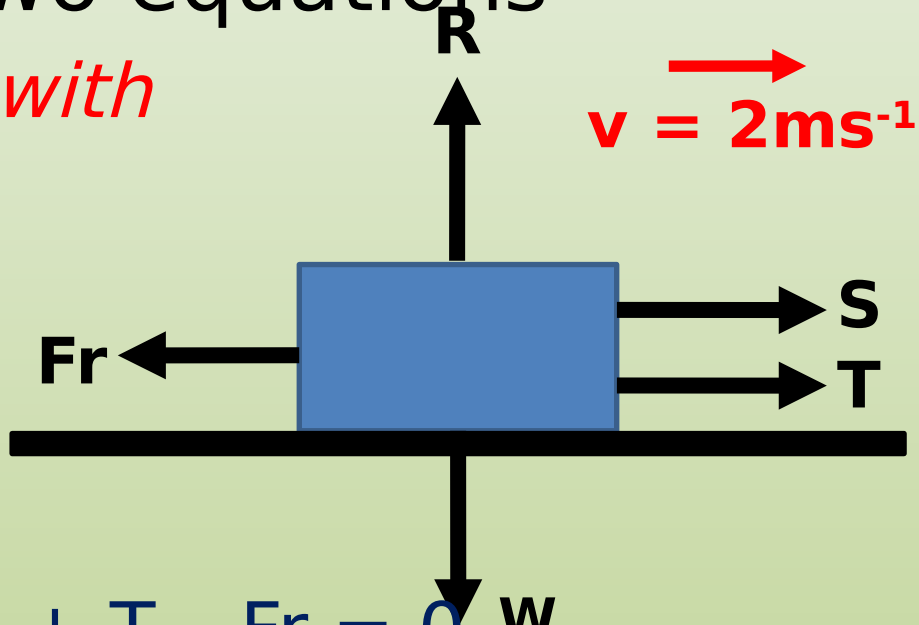


8.1 Forces

Example 4b

b) Resolve forces horizontally and vertically to form two equations

Since the box is moving with constant speed it is in equilibrium, hence the resultant force in any direction must be zero



Resolving horizontally (→): S + T – Fr = 0 ❷

Resolving vertically (↑): R – W = 0 ❷

8.1 Forces

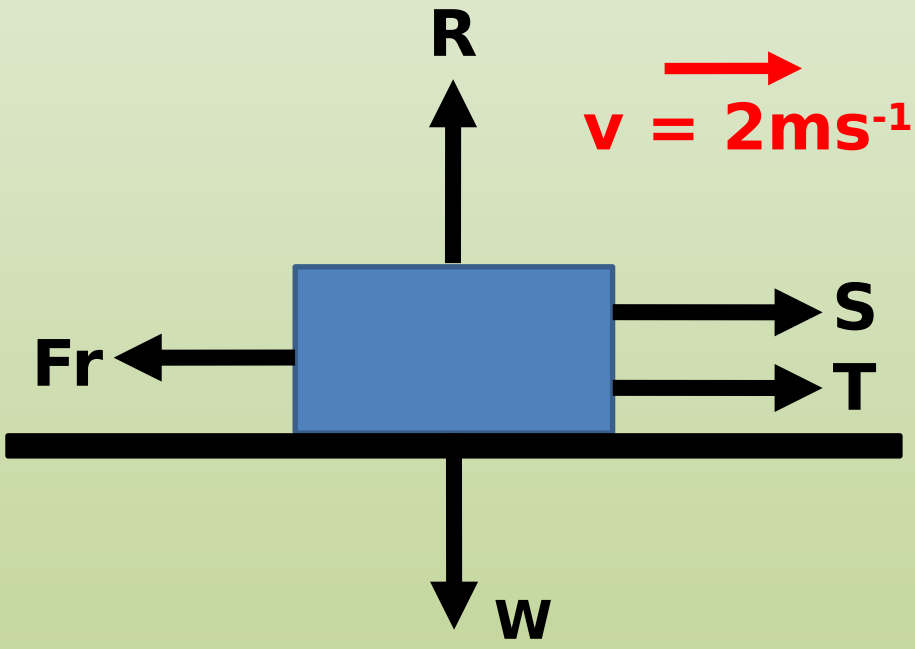
Example 4c

c) If the resistance to motion is 50N and the tension in string S is 15N, find the tension in string T

$S + T - Fr = 0$ ❶

$R - W = 0$ ❷

Using equation ❶:



8.1 Forces

Example 5

An object is held in equilibrium by four forces: $(3\mathbf{i} - 2\mathbf{j})$ N; $(-6\mathbf{i} - 3\mathbf{j})$ N; $(5\mathbf{i} + y\mathbf{j})$ N; $(x\mathbf{i} + 10\mathbf{j})$ N. Find the values of x and y.

Since the object is in equilibrium, the resultant force in any direction must be zero.

$\therefore (3\mathbf{i} - 2\mathbf{j}) + (-6\mathbf{i} - 3\mathbf{j}) + (5\mathbf{i} + y\mathbf{j}) + (x\mathbf{i} + 10\mathbf{j}) = 0\mathrm{i} + 0\mathrm{j}$

$(2 + x)\mathbf{i} + (5 + y)\mathbf{j} = 0\mathbf{i} + 0\mathbf{j}$

Equating coefficients: $\mathbf{i}$: $2 + x = 0 \therefore x = -2$

$\mathbf{j}$: $5 + y = 0 \therefore y = -5$

8.1 Forces

Example 6a

The forces $(3\mathbf{i} - 2\mathbf{j})$ N; $(-6\mathbf{i} - 3\mathbf{j})$ N; $(5\mathbf{i} + 7\mathbf{j})$ N and $(3\mathbf{i} + 10\mathbf{j})$ N act on an object.

a) Calculate the resultant force, in terms of **i** and **j**

$\mathbf{f} = (3\mathbf{i} - 2\mathbf{j}) + (-6\mathbf{i} - 3\mathbf{j}) + (5\mathbf{i} + 7\mathbf{j}) + (3\mathbf{i} + 10\mathbf{j})$

$\mathbf{f} = (3 - 6 + 5 + 3)\mathbf{i} + (-2 - 3 + 7 + 10)\mathbf{j}$

$\therefore \mathbf{f} = (5\mathbf{i} + 12\mathbf{j})$ N

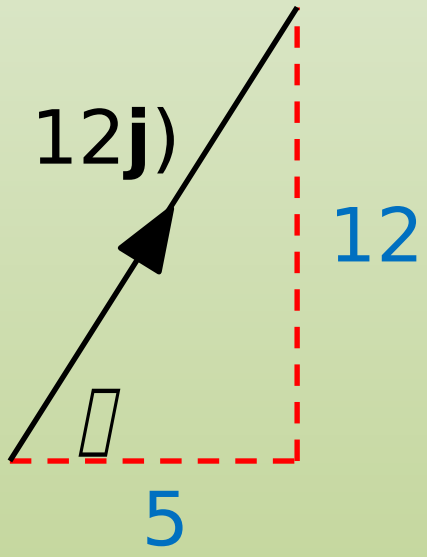
8.1 Forces

Example 6b

The forces (3**i** – 2**j**) N; (-6**i** – 3**j**) N; (5**i** + 7**j**) N and (3**i** + 10**j**) N act on an object.

b) Calculate the magnitude and direction of this resultant force

f = (5**i** + 12**j**) N



8.1 Forces

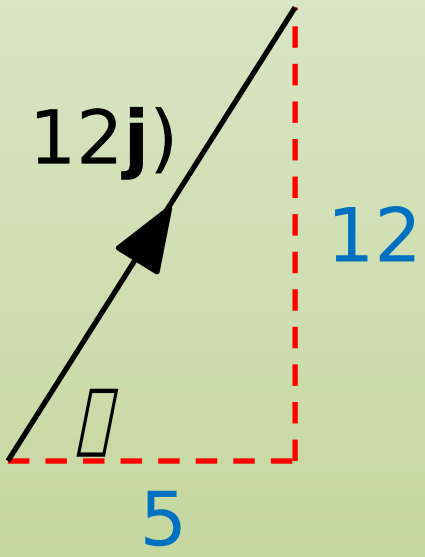
Example 6c

The forces (3**i** – 2**j**) N; (-6**i** – 3**j**) N; (5**i** + 7**j**) N and (3**i** + 10**j**) N act on an object.

c) Describe the motion of the object

The object accelerates in the direction of the resultant force

f = (5**i** + 12**j**) N



8.1 Forces

Example 7

Work out the value of X and Y if the resultant force has a magnitude of 35N at an angle of 30° to the positive x-axis.

Resolving horizontally (→):
50 – X

Resolving vertically (↑):
30 – Y

50 – X = 35cos(30)
X = 50 – 35cos(30) =
19.7N